Probabilistic transitions for P systems*

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Abstract In this paper we use the abstract syntax and the structural operational semantics of the P systems given in [1], and add probabilities to the rules and to the communication targets. We take into account the number of possible combinations of rules which can be applied in a computation step, as well as the consumption degree of the current resources.

Keywords: probabilistic transition, P systems.

The new field of systems biology describes biological processes by using models developed by computer scientists, and tools for simulation and verification of the systems behaviour. Uncertainty (probability theory, fuzzy set theory, rough set theory, approximate reasoning and approximate algorithms) has been used in biology to describe certain biological systems, and to develop software simulators for them[1,2]. There are various attempts of using probabilities in membrane systems[2—4], with applications to biology[5] and economics[6]. The problem of adding probabilities to P systems is still open, and it is considered to be difficult because of various new features introduced by the membrane systems.

In this paper we define the probabilistic transition systems for P systems. The probabilities are added at the level of rules and communication targets. We extend with probabilities the abstract syntax and the structural operational semantic of P system given in [1]. The probabilistic transition system is defined over configurations, and computation is defined as a sequence of probabilistic transitions between configurations. In order to define a probabilistic transition between two configurations of a membrane system, we proceed in three steps:

(i) we define the probabilistic maximal parallel rewriting step consisting in assigning probabilities to rules in every membrane, and executing them in a maximal parallel manner; (ii) then we define the probabilistic parallel communication of objects consisting in assigning probabilities to communication targets, and sending the objects according to these probabilities; (iii) the parallel membrane dissolving step; we do not consider probabilities in this step.

In a previous paper[2], the authors compute (only) the probabilities associated to each applicable rule to the current multiset. Our approach of adding probabilities to a P system and computing them extends this approach, by considering other elements in order to describe the behaviour of a P system: the possible combinations of rules applicable in a computation step, the consumption degree of the current resources in such a computation step of a P system, the probability of sending the messages in the regions specified by the corresponding targets.

1 Preliminaries

1.1 Probabilistic transition system

Since the behaviour of a P system is essentially nondeterministic, we consider systems that can perform both probabilistic and nondeterministic choices. Several types of probabilistic and nondeterministic choices are in the literature. We follow and adapt the approach presented in [7]. Intuitively, a probabilistic and nondeterministic choice is given by sets of alternative transitions, each transition having a certain probability of being selected. The sum of all probabilities of one alternative set must be 1. We represent graphically an alternative set of transitions by using an arc linking its transitions (see Fig. 1). Nondeterminism is given by the fact that only one alternative set of transition is selected, and we have no information on how one alternative is selected. Therefore we have a probabilistic choice inside these alternative sets

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of transitions, while the choice between the alternative sets is nondeterministic.

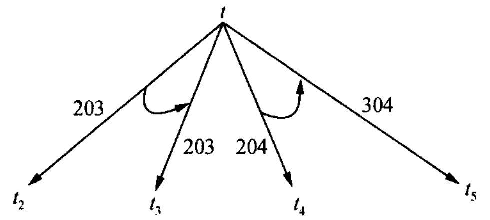

Fig. 1. An alternate set of transitions by using an arc linking its transitions.

Definition 1. A probabilistic transition system for a P system is a tuple $(S, \rightarrow)$, where S is a finite set of states, and $\rightarrow \subseteq S \times [0,1] \times S$ is a probabilistic transition relation. We write $s \xrightarrow{p} t$ instead of $(s, p, t) \in \rightarrow$, where p is the probability to reach t from s. The specific feature is given by the fact that for each $s \in S$ we have alternative sets of transitions given by a partition $S_1, \cdots, S_k$ of S such that $\sum_{t \in S_i} \{p \mid (s, p, t) \in \rightarrow\} = 1$ for all $i = 1, \cdots, k$.

Example 1. The picture above illustrates an example of probabilistic transition system. If s_1, s_2, s_3, and s_4 are the states which can be reached from s with probabilities $\frac{1}{2}, \frac{1}{2}, \frac{1}{3}$, and $\frac{2}{3}$, respectively, then we have a partition of $\{s_1, s_2, s_3, s_4\}$ given by $S' = \{s_1, s_2\}$ and $S'' = \{s_3, s_4\}$ such that $\sum_{t \in S'} \{p \mid (s, p, t) \in \rightarrow\} = 1/2 + 1/2 = 1$ and $\sum_{t \in S''} \{p \mid (s, p, t) \in \rightarrow\} = 1/3 + 2/3 = 1$.

1.2 P systems

Inspired from the cell structure and functioning, the basic elements of a P system are the membrane structure and the sets of evolution rules which process multisets of objects placed in the compartments of the membrane structure. An introduction to the P systems can be found in [8].

Informally a P system consists of several membranes hierarchically embedded in a special membrane called the skin. A membrane without any other membranes inside is elementary, otherwise it is a composite membrane. The membranes produce a demarcation between regions. Regions contain multisets of objects, evolution rules and possibly other membranes. Only rules in a region delimited by a membrane act on the objects in that region. The rules contain target indications, specifying the membrane where the new objects obtained after applying the rule are sent. The new objects either remain in the same region when they have a here target, or they pass through membranes, in two directions: they can be sent out of the membrane delimiting a region from outside, or can be sent in one of the membranes delimiting a region from inside, precisely identified by its label. In a step, the objects can pass only through one membrane. A membrane can contain a δ symbol after a rule application. This means the membrane is dissolved, i.e., it disappears and its contents (objects and membranes) remain in the membrane placed immediately outside, and the evolution rules of the dissolved membranes are lost. The skin membrane is never dissolved (its rules do not involve δ). We also assume the skin does not contain out targets.

Formally a P system is a structure $\Pi = (O, \mu, w_1, \cdots, w_m, (R_1, \rho_1), \cdots, (R_m, \rho_m), i_0)$, where O is the alphabet of objects; μ is the membrane structure; w_i is the initial multiset over O with $i = \overline{1, m}$; R_i is a finite set of evolution rules over O associated with membrane i (the typical form of a rule is $u \rightarrow v$, with u a multiset, and v consisting of a message and/or the dissolving symbol δ); ρ_i is a partial order relation over R_i, specifying a priority relation among the rules: $(r_1, r_2) \in \delta_i$ iff $r_1 > r_2$, i.e., r_1 has a higher priority than r_2; $i_0 \in \{0, 1, \cdots, m\}$ indicate the output membrane of Π (0 being the outer region).

Remark 1. In this paper we describe a multiset w by $w = M(a_1)a_1 \cdots M(a_n)a_n$, where $a_1, \cdots, a_n \in O$ and $M: O \rightarrow \mathbb{N} \setminus \{0\}$ is a mapping which gives the nonzero multiplicities of the objects in O. For example, $w = 2a1c$ is a multiset which contains 2 copies of a and 1 copy of c.

We briefly recall here some notations and concepts related to the operational semantics of P systems given in [1].

1. If X is a set, the free commutative monoid X_c^* denotes the set of the finite multisets defined over X, and $X_c^+ = X_c^* \setminus \{\text{empty}\}$;

2. the mappings rules and priority associate a membrane label to the set of evolution rules and the priority relation: $\text{rules}(L_i) = R_i$, $\text{priority}(L_i) = \rho_i$;

3. the projections L and w return from a membrane its label and its current multiset, respectively;

4. the set of membranes for a P system Π is denoted by $\mathcal{M}(\Pi)$;

5. a simple membrane $\langle L \mid w\rangle \in \mathcal{M}(\Pi)$ has a label L and a multiset of objects $w \in O \cup (O_c^* \times \{\text{here}\}) \cup (O_c^+ \times \{\text{out}\}) \times \{\delta\}$;

6. a composite membrane is $\langle L \mid w; M_1, \cdots, M_n\rangle \in \mathcal{M}(\Pi)$ for a label L and a multiset $w \in O \cup (O_c^* \times \{\text{here}\}) \cup (O_c^+ \times \{\text{out}\}) \cup (O_c^+ \times \{\text{in}_L(M_j)\}) \cup \{\delta\}$;

7. a finite multiset of sibling membranes is written as $M_1, \cdots, M_n$ and it is denoted by M_+; $\mathcal{M}_+(\Pi)$ is the set of non-empty finite multisets of membranes; the union of two multisets of membranes $M_+ = M_1, \cdots, M_m$ and $N_+ = N_1, \cdots, N_n$ is written as $M_+ N_+ = M_1, \cdots, M_m, N_1, \cdots, N_n$;

8. a committed configuration for a P system Π is a skin membrane which has no messages and no dissolving symbol δ; the set of committed configurations for Π is denoted by $C(\Pi) \subset \mathcal{M}^+(\Pi)$;

9. an intermediate configuration for a P system Π is a skin membrane which has messages or the dissolving symbol δ; the set of intermediate configurations for Π is denoted by $C^\#(\Pi) \subset \mathcal{M}^+(\Pi)$;

10. a configuration is either a committed configuration or an intermediate configuration.

2 Probabilistic P systems

There are some previous attempts to assign probabilities to a P system[1–4,9]. Here we come with a new way of computing the probabilities for a computational step in P systems, taking into account all the possible combinations of rules applicable in a computation step. Moreover, we define the consumption degree of the current resources in such a computation step of a P system. In order to do this, we extend the definitions of maximal parallel rewriting relation $\stackrel{mpr}{\Longrightarrow}$ and parallel communication relation $\stackrel{tar}{\Longrightarrow}$ defined in [1] by enriching the inference rules with probabilities, and we denote them by $\xrightarrow{P_{mpr}}_{mpr}$ and $\xrightarrow{P_{tar}}tar$. For consistency, we denote by $\rightarrow_\delta$ the relation for parallel dissolving. In order to get a transition between two configurations of a P system, we follow three steps as in case of P systems without probabilities:

1. The probabilistic maximal parallel rewriting step—consisting in assigning probabilities to rules in every membrane, and executing the rules in a maximal parallel manner;

2. the probabilistic parallel communication of objects step—consisting in assigning probabilities to existing targets, and sending the messages according to these probabilities;

3. the parallel membrane dissolving step—consisting in dissolving the membranes which contain the δ symbol[1]; no probabilities are added.

Note that after applying the first step we get intermediate configurations. After applying the second step we get intermediate configurations if there are δ symbols; otherwise we get committed configurations. If there are δ symbols, then the last step is applied and we get committed configurations. The committed configurations are states of the corresponding probabilistic transition system.

Let Π be a membrane system as described in the previous section. We denote by $\xrightarrow{p} \in C(\Pi) \times [0,1] \times C(\Pi)$ the probabilistic transition relation over the configurations, and by $(C(\Pi), \xrightarrow{p})$ the probabilistic transition system associated to Π. For two configurations C_1, C_2 we write $C_1 \xrightarrow{p} C_2$ if $(C_1, p, C_2) \in \rightarrow$. A transition relation $\xrightarrow{p}$ can be seen as a union of $\xrightarrow{P_{mpr}}mpr$, $\xrightarrow{P_{tar}}tar$ and $\rightarrow_\delta$. The syntax of probabilistic P systems differs from the syntax given in [1] by adding also probabilities to targets. A message contains a multiset of objects w from a given region L, and the pair $(tar, P_{tar}^L(w))$ where $tar \in \{\text{here}, \text{out}, \text{in}_{L'}\}$. This means the probability for sending the multiset w in the region specified by tar is $P_{tar}^L(w)$, and $1 - P_{tar}^L(w)$ otherwise. The set of messages is denoted by $Msg_p(O)$.

Objects:	$o \in O$
Multisets of objects:	$w \in O_c^*$
Labels:	$L \in \{Skin\} \cup \mathscr{L}$
Messages:	$(w, (tar, P_{tar}^{L}(w))) \in Msg_p(O)$,
	$tar \in \{\text{here}, \text{out}, \text{in}_{L'}\}$
Rules:	$\{r_1 : u_1 \xrightarrow{P_L(r_1)} v_1, \cdots, \underline{r_n : u_n} \xrightarrow{P_L(r_n)} v_n\}$
	$0 < P_L(r_i) \leqslant 1,\ i = \overline{1, n}$
Membrane contents:	$w \in (O \cup Msg_p(O) \cup \{\delta\})_c^*$
Membranes:	$M \in \mathcal{M}(\Pi), M ::= \langle L \mid w \rangle \mid \langle L \mid w; M_+ \rangle$
Sibling membranes:	$M_+ \in \mathcal{M}^+(\Pi) = \mathcal{M}(\Pi)_c^+$
Committed configurations:	$C \in C(\Pi)$
Intermediate configurations:	$C \in C^{\#}(\Pi)$

2.1 Probabilistic maximal parallel rewriting

We show here how the probabilities are computed in a maximal parallel rewriting step. The probabilistic maximal parallel rewriting step consists in assigning probabilities to rules in every membrane, and executing them in a maximal parallel manner. Before going into more details, we remind the irreducibility property w.r.t. the maximal parallel rewriting relation for multisets of objects, for membranes, and for sets of sibling membranes[1]. A multiset w of objects is L-irreducible iff there are no rules in rules(L) applicable to w (w.r.t the priority relation priority(L)). A simple membrane $\langle L \mid w \rangle$ is mpr-irreducible iff w is L-irreducible. A non-empty set of sibling membranes $M_1, \cdots, M_n$ is mpr-irreducible if every M_i is mpr-irreducible for every $i = \overline{1, n}$. A composite membrane $\langle L \mid w; M_1, \cdots, M_n \rangle$, with $n \geqslant 1$, is mpr-irreducible iff w is L-irreducible, and the set of sibling membranes $M_1, \cdots, M_n$ is mpr-irreducible.

Let $M = \langle L \mid w; M_1, \cdots, M_l \rangle$ be a composite membrane in $\mathcal{M}(\Pi)$, and the rules applied in membrane L are rules(L) $= \{r_1 : u_1 \to v_1, \cdots, r_n : u_n \to v_n\}$, where $r_i \neq r_j$, for all $i \neq j$. The probability of a rule r_i to be applied is denoted by $P_L(r_i)$; the probability of not applying the rule r_i is $1 - P_L(r_i)$. We write rules(L) as $\{r_1 : u_1 \xrightarrow{P_L(r_1)} v_1, \cdots, r_n : u_n \xrightarrow{P_L(r_n)} v_n\}$.

The consistency of rules for a membrane labelled by L differs from the definition given in [1] by the fact that we add the probability for each rule. Let w be a multiset of objects. A non-empty multiset $R = \{r_1 : u_1 \xrightarrow{P_L(r_1)} v_1, \cdots, r_n : u_n \xrightarrow{P_L(r_n)} v_n\}$ of evolution rules is (L, w)-consistent iff:

1. $R \subseteq$ rules(L),

2. $w = u_1 \cdots u_n z$ such that all the rules in R are applicable in parallel over w with their corresponding probability,

3. $(\forall r \in R, \forall r' \in \text{rules}(L))$ r' applicable on w implies $(r, r') \notin$ priority(L),

4. $(\forall r', r'' \in R)$ $(r', r'') \notin$ priority(L),

5. the dissolving symbol δ has at most one occurrence in the multiset $v_1 \cdots v_n$.

To evaluate the probability for a computation step, we follow three steps:

1. We look for the number of possible combinations of applicable rules in which the objects (resources) can be allocated;

2. compute the probability of each possible combination, and

3. finally attach the consumption degree of the current resources.

The first step represents a combinatorial optimization problem equivalent to knapsack problem (which can be solved in different ways, such as a greedy algorithm, or using dynamic programming[10]).

Since at each step of evaluation all the rules from a combination of applicable rules to the current multiset are simultaneously applied (hence the parallelism is maximal at the level of rules), the probability of choosing a combination is the product of the probabilities of the rules of that combination divided by the number of all the possible combinations. This is expressed in the following definition.

Definition 2. Let $r_1, \cdots, r_n$ be a combination of applicable rules where each r_i applies β_i times, and N is the number of such combinations. Then the probability of choosing this combination of rules is:

$$P(r_1, \cdots, r_n) = \frac{1}{N}\prod_{i=1}^{n}(P_L(r_i))^{\beta_i}$$

Example 2. Assume we have 10 copies of a in a membrane, and the rules inside this membrane are $r_1: 2a \rightarrow b$, $r_2: 3a \rightarrow c$, $r_3: 4a \rightarrow d$, each with the probability $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ respectively. We have 8 possible combinations to apply these rules in a maximal parallel way. The probability to apply, for example, r_1 three times and r_2 once, is $\frac{1}{8}\cdot\left(\frac{1}{2}\right)^3\cdot\frac{1}{3} = \frac{1}{192}$.

Proposition 1. We consider a membrane labelled by L, and its multiset is $w = \gamma_1 a_1 \cdots \gamma_m a_m$, where $\gamma_j \in \mathbb{N} \setminus \{0\}$ is the number of copies of object a_j, $j = \overline{1, m}$. Let $(r_1, \cdots, r_n)$ be a combination of applicable rules over w, β_i the number of applications of rule r_i, and $\alpha_{i_1} a_1 \cdots \alpha_{i_m} a_m$ the left multiset of rule r_i, where $\alpha_{i_j} \in \mathbb{N}$ is the number of copies of object a_j, $j = \overline{1, m}$. The consumption degree of the current resources in membrane L after applying the combination of rules $r_1, \cdots, r_n$, denoted $P_c(r_1, \cdots, r_n)$, is:

$$P_c(r_1, \cdots, r_n) = \frac{C_{\gamma_1}^{\sum_{i=1}^{n}\beta_i\alpha_{i_1}} \cdots C_{\gamma_m}^{\sum_{i=1}^{n}\beta_i\alpha_{i_m}}}{C_{\gamma}^{\sum_{j=1}^{m}\sum_{i=1}^{n}\beta_i\alpha_{i_j}}} \quad (1)$$

where $\gamma = \sum_{j=1}^{m}\gamma_j$ is the total number of objects in membrane L.

Proof. We use notions and results from probability theory[11]. Suppose we have a finite population consisting of N objects distributed as follows: N_1 objects of type 1, $\cdots$, N_k objects of type k, where $\sum_{i=1}^{k} N_i = N$. For example, we could have an urn with balls of several different colours. The event $E =$ "choosing n objects from the population, where n_1 objects are of type 1, $\cdots$, n_k objects are of type k, and $\sum_{i=1}^{k} n_i = n$" follows the multivariate hypergeometric distribution with parameters N, $(N_1, \cdots, N_k)$, and n. The probability of this event is:

$$P(E) = \frac{C_{N_1}^{n_1} \cdots C_{N_k}^{n_k}}{C_N^n}$$

In our case, the population is the membrane with γ objects, where γ_1 objects are of type a_1, $\cdots$, γ_m objects are of type a_m, $\gamma = \sum_{j=1}^{m}\gamma_j$. Applying the rule r_i for β_i times is equivalent to selecting $\beta_i\alpha_{i_j}$ objects of type a_j, $j = \overline{1, m}$.

Example 3. Suppose we have $w = 11a1b5d$, and the rules $r_1: 2a1b \rightarrow b$, $r_2: 3a \rightarrow c$, $r_3: 1a1d \rightarrow d$. The consumption degree of w by applying, for example, r_1 once and r_2 three times is $\frac{C_{11}^{11}C_1^1C_5^0}{C_{17}^{12}} = \frac{1}{6188}$, and by applying r_1 once and r_3 five times is $\frac{C_{11}^{7}C_1^0C_5^5}{C_{17}^{13}} = \frac{33}{238}$. Usually the probability is increasing when the total number of consumed objects is increasing.

Proposition 2. If all the objects in the membrane L are of the same type, say a, and γ is the number of a, i.e. $w = \gamma a$, the consumption degree $P_c(r_1, \cdots, r_n)$ of the current resources in membrane L after applying the combination of rules $r_1, \cdots, r_n$ is

$$P_c(r_1, \cdots, r_n) = \frac{\sum_{i=1}^{n}\beta_i\alpha_i}{\gamma} \quad (2)$$

where the left multiset of rule r_i is equal to $\alpha_i a$, and r_i applies β_i times.

Proof. Let X be the set of possible outcomes of an experiment. Subsets of X are called events. Suppose X is finite, and each of the outcomes in X is equally likely. Then for any event A, $P(A) = \frac{|A|}{|X|}$, where $|A|$ is the number of elements in A, and $|X|$ is the number of elements in X. In our case, $|X|$ is the number of objects in the membrane L, all of the same type. Applying the combination of rules $r_1, \cdots, r_n$, each rule β_i times, the consumption degree of choosing $\sum_{i=1}^{n} \beta_i \alpha_i$ objects is as given in (2).

Example 4. Considering the P system described in Example 2, the consumption degree of $w = 10a$ by applying r_1 three times and r_2 once is $\frac{10}{10} = 1$ because all the resources are consumed, and the probability of applying r_2 three times is $\frac{9}{10}$.

Definition 3. Let have a membrane labelled by L, having w as its multiset, and a set $R = \{r_1, \cdots, r_n\}$ of (L, w)-consistent rules. The probability of a computation step denoted by $P^L_{mpr}(w, w')$ is given by:

$$P^L_{mpr}(w, w') = P((r_{i_1} \times \beta_{i_1}, \cdots, r_{i_k} \times \beta_{i_k})) \cdot P_c((r_{i_1} \times \beta_{i_1}, \cdots, r_{i_k} \times \beta_{i_k})) \quad (3)$$

where N is the number of possible combinations to apply the rules $r_1, \cdots, r_n$, $P(r_{i_1}, \cdots, r_{i_k})$ is the probability of choosing the combination $(r_{i_1}, \cdots, r_{i_k})$, with $i_1, \cdots, i_k \in \{1, \cdots, n\}$, and $P_c(r_{i_1}, \cdots, r_{i_k})$ is the consumption degree of the resources in membrane L by applying this combination.

We write $w \xrightarrow{P^L_{mpr}}_{mpr, L} w'$.

Definition 4. Let $M, M' \in \mathcal{M}(\Pi)$, where $M = \langle L \mid w; M_1, \cdots, M_l \rangle$, $M \stackrel{mpr}{\Longrightarrow} M'$, and $M' = \langle L \mid w'; M'_1, \cdots, M'_l \rangle$. We denote by $P^c_{mpr}(M, M')$ the proba-bility of reaching M' by applying the rules in membrane M. We have:

$$P^c_{mpr}(M, M') = P^L_{mpr}(w, w') \prod_{i=1}^{l} P^c_{mpr}(M_i, M'_i) \quad (4)$$

We write $M \xrightarrow{P^c_{mpr}}_{mpr} M'$.

Proposition 3. For $M_+ \in \mathcal{M}^+(\Pi)$, $M_+ = M_1, M_2, \cdots, M_l$, $M_+ \stackrel{mpr}{\Longrightarrow} M'_+$, and $M'_+ \in \mathcal{M}^+(\Pi)$, $M'_+ = M'_1, M'_2, \cdots, M'_l$, the probability of reaching M'_+ by applying the rules in M_+ is:

$$P^+_{mpr}(M_+, M'_+) = \prod_{i=1}^{l} P^c_{mpr}(M_i, M'_i) \quad (5)$$

We write $M_+ \xrightarrow{P^+_{mpr}}_{mpr} M'_+$.

Proof. The proof follows by induction on the structure of membranes $M_1, M_2, \cdots, M_l$, and by applying the previous definitions (3) and (4).

Remark 2. If not all M_i are mpr-irreducible, then

$$P^+_{mpr}(M_+) = \prod_{i=1}^{l} \{P^c_{mpr}(M_i) \mid M_i \text{ is not } mpr\text{-irreducible}\} \quad (6)$$

Now we can define the maximal parallel rewriting relations $\xrightarrow{P_{mpr}}_{mpr, L}$ and $\xrightarrow{P_{mpr}}_{mpr}$ by the following inference rules:

For each $w = u_1 \cdots u_n z \in O_c^+$ such that z is L-irreducible, and (L, w)-consistent rules $(u_1 \xrightarrow{P_L(r_1)} v_1, \cdots, u_n \xrightarrow{P_L(r_n)} v_n)$,

$$(\mathbf{R}_1) \quad \frac{}{u_1 \cdots u_n z \xrightarrow{P^L_{mpr}}_{mpr, L} v_1 \cdots v_n z}$$

where $P^L_{mpr}(w, v_1 \cdots v_n z)$ is defined as in (3).

For each $w \in O_c^+$, $w' \in (O \cup Msg_p(O) \cup \{\delta\})_c^+$ and label L,

$$(\mathbf{R}_2) \quad \frac{w \xrightarrow{P^L_{mpr}}_{mpr, L} w'}{\langle L \mid w \rangle \xrightarrow{P^L_{mpr}}_{mpr} \langle L \mid w' \rangle}$$

For each $w \in O_c^+$, $w' \in (O \cup Msg_p(O) \cup \{\delta\})_c^+$, $M_+, M'_+ \in \mathcal{M}^+(\Pi)$, and label L,

$$(\mathbf{R}_3) \quad \frac{w \xrightarrow{P^L_{mpr}}_{mpr, L} w', \ M_+ \xrightarrow{P^+_{mpr}}_{mpr} M'_+}{\langle L \mid w; M_+ \rangle \xrightarrow{P^c_{mpr}}_{mpr} \langle L \mid w'; M'_+ \rangle}$$

where $P^c_{mpr}(\langle L \mid w; M_+ \rangle, \langle L \mid w'; M'_+ \rangle) = P^L_{mpr}(w, w') P^+_{mpr}(M_+, M'_+)$.

For each $w \in O_c^+$, $w' \in (O \cup Msg_p(O) \cup \{\delta\})_c^+$, mpr-irreducible $M_+ \in \mathcal{M}^+(\Pi)$, and label L,

$$(\mathbf{R}_4) \quad \frac{w \xrightarrow{P^L_{mpr}}_{mpr, L} w'}{\langle L \mid w; M_+ \rangle \xrightarrow{P^c_{mpr}}_{mpr} \langle L \mid w'; M_+ \rangle}$$

where $P^c_{mpr}(\langle L \mid w; M_+ \rangle, \langle L \mid w'; M_+ \rangle) = P^L_{mpr}(w, w')$.

For each L-irreducible $w \in O_c^*$, and M_+, M'_+

$\in \mathcal{M}^+(\Pi)$, and label L,

$$(\mathbf{R}_5)\ \frac{M_+ \xrightarrow{P^+_{mpr}}_{mpr} M'_+}{\langle L \mid w; M_+\rangle \xrightarrow{P^c_{mpr}}_{mpr} \langle L \mid w; M'_+\rangle}$$

where $P^c_{mpr}(\langle L \mid w; M_+\rangle, \langle L \mid w; M'_+\rangle) = P^+_{mpr}(M_+, M'_+)$.

For each $M, M' \in \mathcal{M}(\Pi)$, and $M_+, M'_+ \in \mathcal{M}^+(\Pi)$,

$$(\mathbf{R}_6)\ \frac{M \xrightarrow{P^c_{mpr}}_{mpr} M',\ M_+ \xrightarrow{P^+_{mpr}}_{mpr} M'_+}{M, M_+ \xrightarrow{P^{+'}_{mpr}}_{mpr} M', M'_+}$$

where $P^{+'}_{mpr}((M, M_+), (M', M'_+)) = P^c_{mpr}(M, M')P^+_{mpr}(M_+, M'_+)$.

For each $M, M' \in \mathcal{M}(\Pi)$, and mpr-irreducible $M_+, \in \mathcal{M}^+(\Pi)$,

$$(\mathbf{R}_6)\ \frac{M \xrightarrow{P^c_{mpr}}_{mpr} M'}{M, M_+ \xrightarrow{P^{+'}_{mpr}}_{mpr} M', M_+},$$

where $P^{+'}_{mpr}((M, M_+), (M', M_+)) = P^c_{mpr}(M, M')$.

The related results given in [1] hold here as well.

2.2 Probabilistic parallel communication

After a probabilistic maximal parallel rewriting step, the resulted multisets contain messages with targets and probabilities of placing these multisets in the regions indicated by the targets. In this section we evaluate the probability of getting new intermediate or committed configurations after sending these messages. We first recall some notions regarding P systems without probabilities. We say that a multiset w inside a membrane L is *here-free*/*in*$_{L'}$-*free*/*out-free* if it does not contain any *here*/*in*$_L$/*out* messages, respectively[1]. For a multiset w of objects and messages, the operations *obj*, *here*, *out*, *in*$_{L'}$ are defined as follows:

obj(w) is obtained from w by removing all the messages.

$$here(w) = \begin{cases} empty, & \text{if } w \text{ is } here\text{-}free \\ w'', & \text{if } w = w'(w'', P^L_{here}(w'')) \wedge P^L_{here}(w'') \neq 0 \wedge w' \ out\text{-}free \end{cases}$$

$$out(w) = \begin{cases} empty, & \text{if } w \text{ is } out\text{-}free \\ w'', & \text{if } w = w'(w'', P^L_{out}(w'')) \wedge P^L_{out}(w'') \neq 0 \wedge w' \ out\text{-}free \end{cases}$$

$$in_{L'}(w) = \begin{cases} empty, & \text{if } w \text{ is } in_{L'}\text{-}free \\ w'', & \text{if } w = w'(w'', P^L_{in_{L'}}(w'')) \wedge P^L_{in_{L'}}(w'') \neq 0 \wedge w' \ in_{L'}\text{-}free \end{cases}$$

The definition of tar-irreducibility property is given as in [1]. A simple membrane $\langle L \mid w\rangle$ is tar-irreducible iff $L = Skin \vee (L \neq Skin \wedge w$ is *out-free*). A non-empty set of sibling membranes $M_1, \cdots, M_n$ is tar-irreducible iff M_i is tar-irreducible, for every $i = \overline{1, n}$. A composite membrane $\langle L \mid w; M_1, \cdots, M_n\rangle$ with $n \geqslant 1$ is tar-irreducible iff:

1. $L = Skin \vee (L \neq Skin \wedge w$ is *out-free*),

2. w is $in_{L(M_i)}$-*free*, for every $i = \overline{1, n}$,

3. for all $i = \overline{1, n}$, $w(M_i)$ is *out-free*,

4. the set of sibling membranes $M_1, \cdots, M_n$ is tar-irreducible.

Since at each step of evaluation, all the existing messages from a multiset w are sent simultaneously in the regions indicated by the targets, the probability of sending all the messages of w is the product of all probabilities of placing the multisets with targets in the corresponding regions. This is expressed in the following definition.

Definition 5. The probability of sending all the messages of w in the regions indicated by their targets is $P^L_{tar}(w)$, where $w = (v_1, (tar_1, P^L_{tar}(v_1))), \cdots, (v_n, (tar_n, P^L_{tar}(v_n)))z$, $v_i \in O_c^*$, $i = \overline{1, n}$ and z is tar-irreducible.

$$P^L_{tar}(w) = \prod_{i=1}^{n} P^L_{tar_i}(v_i) \qquad (7)$$

Definition 6. Let $M, M' \in \mathcal{M}(\Pi)$, where $M = \langle L \mid w; M_1, \cdots, M_l\rangle$, $M \stackrel{tar}{\Rightarrow} M'$, and $M' = \langle L \mid w'; M'_1, \cdots, M'_l\rangle$. We denote by $P^c_{tar}(M, M')$ the probability of reaching M' by sending the existing messages from M. Then we have:

$$P^c_{tar}(M, M') = P^L_{tar}(w)\prod_{i=1}^{l} P^c_{tar}(M_i, M_i') \quad (8)$$

We write $M \xrightarrow{P^c_{tar}}_{tar} M'$.

Proposition 4. For $M_+ \in \mathcal{M}^+(\Pi)$, $M_+ = M_1, M_2, \cdots, M_l$, $M_+ \stackrel{tar}{\Rightarrow} M'_+$, and $M'_+ \in \mathcal{M}^+(\Pi)$, $M'_+ = M'_1, M'_2, \cdots, M'_l$, the probability of reaching M'_+ by sending the existing messages from M_+ is

$$P^+_{tar}(M_+, M'_+) = \prod_{i=1}^{l} P^c_{tar}(M_i, M_i') \quad (9)$$

We write $M_+ \xrightarrow{P^+_{tar}}_{tar} M'_+$.

Proof. The proof follows by induction on the structure of composite membranes $M_1, M_2, \cdots, M_l$, and by applying Definition 6.

Remark 3. If not all M_i are tar-irreducible, then

$$P^+_{tar}(M_+) = \prod_{i=1}^{l} \{P^c_{tar}(M_i) \mid M_i \text{ is not tar-irreducible}\} \quad (10)$$

Now we can define the parallel communication relation $\xrightarrow{P_{tar}}_{tar}$ by the following inference rules:

For each tar-irreducible $M_1, \cdots, M_n \in \mathcal{M}^+(\Pi)$, label L, and multisets w such that $here(w) \neq empty$ or $L \neq Skin \wedge out(w) \neq empty$, or there exists $i = \overline{1, n}$ with $in_{L(M_i)}(w)out(w(M_i)) \neq empty$, or $here(w(M_i)) \neq empty$,

$$(\mathbf{C}_1)\ \frac{}{\langle L \mid w; M_1, \cdots, M_n\rangle \xrightarrow{P^c_{tar}}_{tar} \langle L \mid w'; M'_1, \cdots, M'_n\rangle}$$

where $P^c_{tar}(M, M')$ is defined as in (8), M and M' are on the left and right hand sides of the rule conclusion, $w' = obj(w)here(w)out(w(M_1))\cdots out(w(M_n))$ and $w(M'_i) = obj(w(M'_i))here\cdot(w(M'_i))in_{L(M_i)}(w)$, for all $i = \overline{1, n}$.

For each $M_+ = M_1, \cdots, M_n$, $M'_+ = M'_1, \cdots, M'_n \in \mathcal{M}^+(\Pi)$, multiset w, and label L,

$$(\mathbf{C}_2)\ \frac{M_1, \cdots, M_n \xrightarrow{P^+_{tar}}_{tar} M'_1, \cdots, M'_n}{\langle L \mid w; M_1, \cdots, M_n\rangle \xrightarrow{P^c_{tar}}_{tar} \langle L \mid w''; M''_1, \cdots, M''_n\rangle}$$

where

$$P^c_{tar}(M, M'') = P^L_{tar}(w)P^+_{tar}(M, M')$$

M and M'' are on the left and right hand side of the rule conclusion

$w'' = obj(w)here(w)out(w(M'_1))\cdots out(w(M'_n))$
$w(M''_i) = obj(w(M'_i))here(w(M'_i))in_{L(M_i)}(w)$,
for all $i = \overline{1, n}$.

For each multiset w such that $here(w)out(w) \neq empty$,

$$(\mathbf{C}_3)\ \frac{}{\langle Skin \mid w\rangle \xrightarrow{P^c_{tar}}_{tar} \langle Skin \mid obj(w)here(w)\rangle}$$

where $P^c_{tar}(M, M') = P^{Skin}_{tar}(w)$, and $M = \langle Skin \mid w\rangle$ and $M' = \langle Skin \mid obj(w)here(w)\rangle$.

For each $M, M' \in \mathcal{M}(\Pi)$, and tar-irreducible $M_+ \in \mathcal{M}^+(\Pi)$,

$$(\mathbf{C}_4)\ \frac{M \xrightarrow{P^c_{tar}}_{tar} M'}{M, M_+ \xrightarrow{P^+_{tar}}_{tar} M', M_+}$$

where $P^+_{tar}((M, M_+), (M', M_+)) = P^c_{tar}(M, M')$.

For each $M \in \mathcal{M}(\Pi)$, $M_+ \in \mathcal{M}^+(\Pi)$,

$$(\mathbf{C}_5)\ \frac{M \xrightarrow{P^c_{tar}}_{tar} M',\ M_+ \xrightarrow{P^+_{tar}}_{tar} M'_+}{M, M_+ \xrightarrow{P^{+'}_{tar}}_{tar} M', M_+}$$

where $P^{+'}_{tar}((M, M_+), (M', M'_+)) = P^c_{tar}(M, M')P^+_{tar}(M'_+)$.

2.3 Probabilistic transition system for a P system

Let Π be a membrane system. The probabilistic operational semantics of Π corresponds to a transition system where states are identified with configurations. Let $(C(\Pi), \xrightarrow{p})$ be the probabilistic transition system associated to Π, and $C_1, C_2 \in C(\Pi)$ be two configurations of the probabilistic transition system. The transition $\xrightarrow{p}$ can be seen as a union of $\xrightarrow{P_{mpr}}_{mpr}$, $\xrightarrow{P_{tar}}_{tar}$ and $\rightarrow_\delta$. The probability to reach C_2 from C_1 is computed using conditional probabilities. Let define the following events:

$A = \{$reaching C'_1 starting from C_1 by applying $\xrightarrow{p}_{mpr}\}$, and

$B = \{$reaching C_2 starting from C'_1 by applying $\xrightarrow{p}_{tar}\}$.

Then the probability to reach C_2 from C_1 is given by

$$\widetilde{P}(C_1, C_2) = P(A \text{ and } B) = P(B \mid A)P(A)$$
$$= P^c_{tar}(C_1', C_2)P^c_{mpr}(C_1, C_1'),$$

where $C_1' \in C^{\#}(\Pi)$ and $C_2 \in C(\Pi)$.

Since the rules from each membrane are applied in a maximal parallel way, the cumulative probability could be greater than 1 or could be approximately 0. For that we have to do a normalization in order to obtain probabilistic and nondeterministic system. Let $C_{11}, \cdots, C_{1n}$ be the states that can be reached from C_1. Then, the probability to reach a state C_{1i} is:

$$p = P(C_1, C_{1i}) = \frac{\widetilde{P}(C_1, C_{1i})}{\sum_{i=1}^{n} P(C_1, C_{1i})}$$

Since we get a probabilistic transition system, we can apply the existing methods for formal verification of probabilistic P systems[7,12]. In order to analyze a probabilistic model which has been specified, it is necessary to identify one or more properties of the model which can be evaluated by a tool. While some properties can be studied in a non-probabilistic setting, others, such as system performance and reliability, steady-state analysis, and so on, require a probabilistic description of the system. As tools of verification we can use PRISM[13], a probabilistic model-checker is available at http:// www.cs.bham.ac.uk/~dxp/prism. Typically for expressing the properties checked by PRISM we use a temporal logic.

3 Conclusion and further work

In this paper we defined the probabilistic operational semantics of a P system by adding probabilities at the level of rules and targets. Essentially, we extend with probabilities the abstract syntax and the structural operational semantic of P system given in [1]. A specific feature of this approach is given by the combinatorial nature of these probabilities. The way of adding and computing the probabilities of a membrane system extends the method presented in [2] for a dynamical probabilistic P systems, where the probabilities associated to rules change during the evolution of the system.

Some attempts to address these topics in membrane computing have been reported. One is in [3], where M. Madhu defines a variant of P systems called probabilistic rewriting P systems, where the selection of rewriting rules is probabilistic. The author shows that, with nonzero cut-point, probabilistic rewriting P systems with/without priorities generate only finite languages, but with zero cut-point and without priorities, probabilistic rewriting P systems of degree 1 characterize the family of languages generated by matrix grammars.

A different approach is chosen in [9], where stochastic P systems are introduced by starting from stochastic Petri nets, and randomized P systems are used to implement randomized algorithms which can solve hard problems in linear time with a high enough probability, making use of a sub-exponential workspace. Obtulowicz discusses in [4] various ways of associating probabilities in membrane computing: to single objects, to multiplicities of objects (hence to multisets), to rules (depending or not on the previous applied rules), to the communication targets.

Interesting applications of the probabilistic P systems can be in describing biological processes. Such an approach is presented in [5]. The probabilistic P systems can also model various aspects in economy, as it is already suggested in [6].

There exist many variants and classes of P systems; many of them are introduced in [8]. We intend to study the probabilistic approach of some interesting classes of P systems. We also work to a probabilistic simulator P system based on the syntax and semantic of the P system given in this paper. The aim is to get some interesting results, and good graphical representations for the behaviour of a probabilistic P system. For example, we can compute the probability that a certain object is present in a specified configuration.

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